5.1

Direct Variation

The distance that a person can jog is related to time. If you are jogging at a constant speed of 100 m/min, how far can you jog in 10 min? in 1 h?

Tools

- grid paper

Making Connections

You learned about independent and dependent variables in Chapter 2: Relations.

Investigate

What is the relationship between distance and time?

1. Susan can jog at a steady pace of 150 m/min for the first hour.

a) Create a table showing the distance that Susan jogs in 0 min, 1 min, 2 min, and so on up to 10 min.

b) Identify the **independent variable** and the **dependent variable**. Graph this relationship.

c) Describe the shape of the graph. Where does it intersect the vertical axis?

d) Write an equation to find the distance, d, in metres, that Susan jogs in t minutes.

e) Use the equation to determine the distance that Susan can jog in 40 min.

f) Consider the distance Susan jogged in 2 min. What happens to the distance when the time is doubled? What happens to the distance when the time is tripled?

2. Trish's steady jogging pace is 175 m/min. Repeat step 1 using Trish's speed.

3. **Reflect** Describe how to develop an equation for distance when you know the average speed.

The relationship between distance and time is an example of a **direct variation**. For example, the table shows distances travelled in various time periods at a constant speed of 10 m/s.

Time (s)	Distance (m)
1	10
2	20
3	30
4	40
5	50

When time is multiplied by a specific number, distance is also multiplied by the same number. Another way to describe this direct variation is to say that distance *varies directly* with time.

In a direct variation, the ratio of corresponding values of the variables does not change. So, if d is distance and t is time, then $\frac{d}{t} = k$, where k is called the **constant of variation**. Multiplying both sides of the equation by t gives $d = kt$.

For the data in the table, $\frac{d}{t} = 10$, or $d = 10t$. The constant of variation is 10.

direct variation

- a relationship between two variables in which one variable is a constant multiple of the other

constant of variation

- in a direct variation, the ratio of corresponding values of the variables, often represented by k, or the constant multiple by which one variable is multiplied
- if d varies directly as t, then the constant of variation, k, is given by

$k = \frac{d}{t}$ or $d = kt$

Example 1 Algebraic Direct Variation

The Fredrick family travels 250 km to a relative's home. The distance, d, in kilometres, varies directly with the time, t, in hours.

a) Find the equation relating d and t if $d = 43$ when $t = 0.5$. What does the constant of variation represent?

b) Use the equation to determine how long it will take the Fredricks to reach their destination.

Solution

a) Since d varies directly with t, the equation has the form $d = kt$.

To find k, substitute the given values into $k = \frac{d}{t}$.

$$k = \frac{43}{0.5}$$
$$= 86$$

The constant of variation represents the constant average speed, 86 km/h. The equation relating d to t is $d = 86t$.

b) Substitute $d = 250$.

$$250 = 86t$$
$$\frac{250}{86} = t \qquad \text{Divide both sides by 86.}$$
$$2.9 \doteq t$$

It will take the Fredricks about 2.9 h to reach their destination.

Example 2 Hourly Rate of Pay

Amir works part-time at a local bookstore. He earns \$7.50/h.

a) Describe the relationship between his pay, in dollars, and the time, in hours, he works.

b) Illustrate the relationship graphically and represent it with an equation.

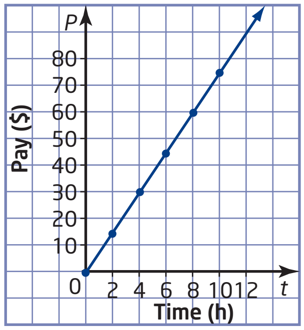
c) One week, Amir works 9 h. Find his pay for that week.

Solution

a) To get Amir's pay, multiply the time worked, in hours, by \$7.50. This means that Amir's pay, P, in dollars, varies directly with the time, t, in hours worked.

b) Method 1: Pencil and Paper

Time Worked, t (h)	Pay, P (\$)
0	0
2	15
4	30
6	45
8	60
10	75

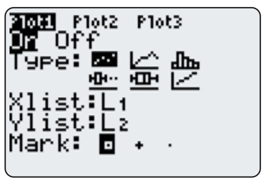


This direct variation can be modelled by the equation $P = 7.50t$, where $k = 7.50$ is the constant of variation.

Method 2: Use a Graphing Calculator

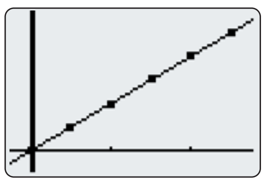
Use the data from the table in **Method 1**.

- To clear all lists, press 2nd [MEM] to display the **MEMORY** menu, select **4:ClrAllLists**, and press ENTER.
- To enter the data into the lists, press STAT and select **1:Edit**. Under list **L1**, enter the values for time worked, in hours. Under list **L2**, enter the values for pay, in dollars.
- To display the scatter plot, set up Plot1 as shown. Press ZOOM and select **9:ZoomStat**.

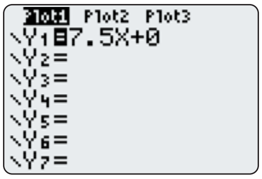


Draw the line of best fit.

- Press STAT, cursor over to display the **CALC** menu, and then select **4:LinReg(ax+b)**. Enter **L1**, a comma, **L2**, and another comma. Then, press VARS, cursor over to display the **Y-VARS** menu, then select **1:FUNCTION**, and then **1:Y1**. Press ENTER, and then press GRAPH.



• Press (Y=) to see the equation representing the relationship between the time, in hours, worked and Amir's pay, in dollars.
Y1 = 7.50X



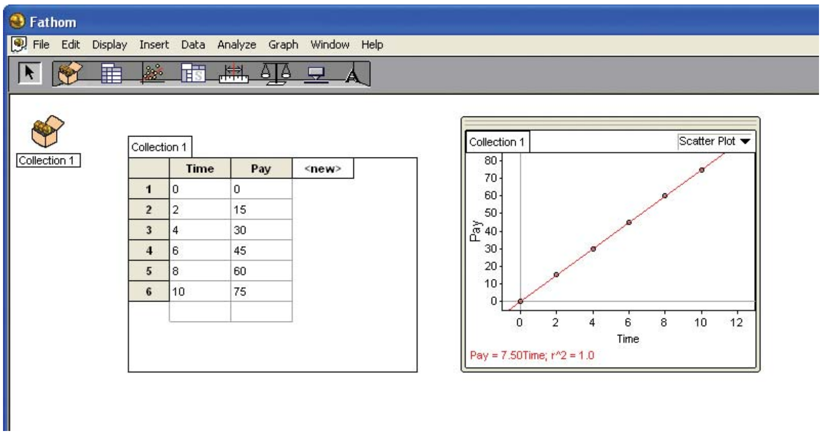
Method 3: Use *Fathom*™

From the **Tool shelf**, click and drag the **Case Table** icon into the workspace. Name two attributes Time and Pay. Enter the data from the table in **Method 1** into the appropriate cells.

From the **Tool shelf**, click and drag the **New Graph** icon into the workspace. Drag the Time attribute to the horizontal axis and the Pay attribute to the vertical axis. You will see a scatter plot of the data.

From the **Graph** menu, select **Least-Squares Line**.
The equation representing the relationship between the time worked and Amir's pay will be indicated in the space below the graph.

Pay = 7.50Time



c) Interpolate from the graph. Read up from 9 h on the horizontal axis to the line. Then, read across to find that Amir's pay is about \$68.

You can also use the equation. Substitute $t = 9$ into $P = 7.50t$.

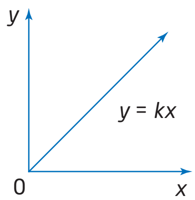
$$P = 7.50(9)$$
$$= 67.50$$

In this case, if I use the graph, I only get an approximate answer, but if I use the equation, I get an exact answer.

Amir's pay for 9 h is \$67.50.

Key Concepts

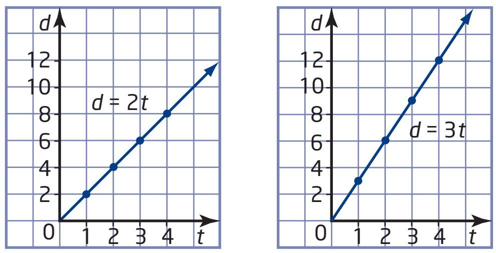
- Direct variation occurs when the dependent variable varies by the same factor as the independent variable.
- Direct variation can be defined algebraically as $\frac{y}{x} = k$ or $y = kx$, where k is the constant of variation.
- The graph of a direct variation is a straight line that passes through the origin.



Communicate Your Understanding

C1 Consider the two equations $A = 2C + 5$ and $A = 2C$. Which is an example of a direct variation? Explain.

C2 Consider the graphs of $d = 2t$ and $d = 3t$.



a) Describe the similarities.

b) Describe the differences. Explain why these differences occur.

Practise

For help with questions 1 and 2, see Example 1.

1. Determine the constant of variation for each direct variation.

a) The distance travelled by a bus varies directly with time. The bus travels 240 km in 3 h.

b) The total cost varies directly with the number of books bought. Five books cost \$35.

c) The volume of water varies directly with time. A swimming pool contains 500 L of water after 5 min.

2. The cost, C, in dollars, of building a concrete sidewalk varies directly with its length, s, in metres.

a) Find an equation relating C and s if a 200-m sidewalk costs \$4500.

b) What does the constant of variation represent?

c) Use the equation to determine the cost of a 700-m sidewalk.

For help with questions 3 to 5, see Example 2.

3. Passent's pay varies directly with the time, in hours, she works. She earns \$8/h.

a) Choose appropriate letters for variables. Make a table of values showing Passent's pay for 0 h, 1 h, 2 h, and 3 h.

b) Graph the relationship.

c) Write an equation in the form $y = kx$.

4. The total cost of apples varies directly with the mass, in kilograms, bought. Apples cost \$1.50/kg.

a) Choose appropriate letters for variables. Make a table of values showing the cost of 0 kg, 1 kg, 2 kg, and 3 kg of apples.

b) Graph the relationship.

c) Write an equation in the form $y = kx$.

5. A parking garage charges \$2.75/h for parking.

a) Describe the relationship between the cost of parking and the time, in hours, parked.

b) Illustrate the relationship graphically and represent it with an equation.

c) Use your graph to estimate the cost for 7 h of parking.

d) Use your equation to determine the exact cost for 7 h of parking.

Connect and Apply

6. The cost of oranges varies directly with the total mass bought. 2 kg of oranges costs \$4.50.

a) Describe the relationship in words.

b) Write an equation relating the cost and the mass of the oranges. What does the constant of variation represent?

c) What is the cost of 30 kg of oranges?

7. To raise money for a local charity, students organized a wake-a-thon where they attempted to stay awake for 24 h. At this event, the amount of money raised varied directly with the time, in hours, a participant stayed awake. Tania raised $50 by staying awake for 16 h.

a) Graph this direct variation for times from 0 h to 16 h, using pencil and paper or technology.

b) Write an equation relating the money Tania raised and the amount of time, in hours, she stayed awake.

c) How much would she have raised by staying awake for 24 h?

8. At his summer job, Sam's regular wage is $9.50/h. For any overtime, Sam earns 1.5 times his regular wage.

a) Write an equation representing Sam's regular pay.

b) Write a separate equation representing Sam's overtime pay.

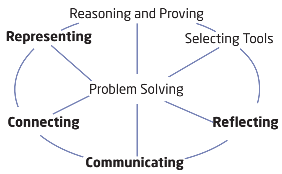
c) Sam gets a raise to $10/h. How does this change affect the equations?

9. At a bulk store, 0.5 kg of sugar costs $1.29.

a) Explain why this relationship is considered a direct variation.

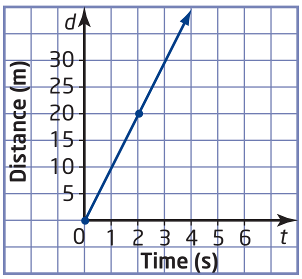
b) Graph this relationship, using pencil and paper or technology.

c) What would happen to the graph if the price increased to $1.49 for 0.5 kg?

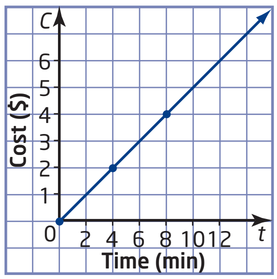


10. Describe a situation that could be illustrated by each graph.

a)



b)



11. A bat uses sound waves to avoid flying into objects. A sound wave travels at 342 m/s. The times for sound waves to reach several objects and return to the bat are shown in the table. Set up an equation to determine the distance from the bat to the object. Then, copy and complete the table.

Object	Time (s)	Distance (m)
Tree	0.1	
House	0.25	
Cliff wall	0.04	

12. The volume of water in a swimming pool varies directly with time. 500 L of water is in the pool after 4 min.

a) Write an equation relating the volume of water and time. What does the constant of variation represent?

b) Graph this relationship using pencil and paper or technology.

c) What volume of water is in the swimming pool after 20 min?

d) How long will it take to fill a swimming pool that holds 115 000 L of water?

e) Describe the changes to the equation and graph if only 400 L of water is in the pool after 4 min.

13. The freezing point of water varies directly with the salt content of the water. Fresh water (no salt content) freezes at a temperature of 0°C. Ocean water has a salt content of 3.5% and freezes at −2°C.

a) Which is the independent variable? Why?

b) Write an equation relating the freezing point of water and the salt content.

c) At what temperature will water with a salt content of 1% freeze?

d) What is the salt content of water that freezes at −3°C?

Extend

14. To convert from kilometres to miles, multiply by 0.62. Write an equation to convert miles to kilometres.

15. Determine the set of ordered pairs that lists the diameter and circumference of four different coins: a penny, a nickel, a dime, and a quarter. Does the circumference vary directly with the diameter? Explain.

16. **Math Contest** From a bag of disks numbered 1 through 100, one disk is chosen. What is the probability that the number on the disk contains a 3? Justify your answer.

17. **Math Contest** The digits 2, 3, 4, 5, and 6 are used to create five-digit odd numbers, with no digit being repeated in any number. Determine the difference between the greatest and least of these numbers.